# Floquet-Bloch waves in periodic chiral media

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The Floquet-Bloch waves in periodically inhomogeneous chiral media are considered. The wave vectors are assumed to be arbitrarily oriented with respect to the grating vector. We have formulated general equations of the dynamical diffraction by such a medium and have analyzed them in the two-wave approximation under different conditions. It is shown that the band structure of the eigenmode spectrum can be qualitatively modified under the effect of chirality. At a sufficiently large chiral admittance, the band gap that is forbidden for a given circularly polarized wave is split into two—central band gap and chiral satellite—which are separated by an interval of transparency. The bandwidth of the satellite is strongly dependent on the propagation angle and approaches zero as the wave vectors tend to be collinear with the grating vector. A change from the conventional Bragg diffraction band structure to the complex-split one is through the range where points of $J$ multiplicity and anomalous dispersion are characteristic of the spectrum. The boundary-value problem under diffraction by a periodic chiral layer is also analyzed. A unified expression for the reflection coefficient of the layer suitable over the entire region of the two-wave diffraction was obtained in the case of the complex-split spectrum. It is shown that the presence of satellites in the spectrum results in the appearance of additional reflection band gaps spaced both above and below in frequency relative to the central band gap. Unlike the latter, the side band gaps each are opaque for a given circularly polarized component of an incident field whereas the oppositely polarized component passes through the layer without Bragg reflection.

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## I. INTRODUCTION

Chiral media have been extensively studied in recent years. They play an important role in applications in a variety of fields such as radiophysics, optics, biology, and chemistry. Electrodynamics of chiral media has been the objective of monographs [1,2]. An approach based on Beltrami-Maxwell field formalism [1] and covariant technique [3,4] proves to be highly efficient as applied to the description of such media. But properties of periodically inhomogeneous chiral media have yet received little attention.

Highly general methods, both rigorous numerical and approximate analytical, are well developed in a theory of periodical media [5–16]. These methods have been successfully applied to structures of different kinds, such as antennas and microwave waveguides [5–8], photonics crystals [9], optical gratings [10–15], and crystals under x-ray diffraction [13,16]. All these methods are based on either the Bloch function formalism or the coupled wave equation approach.

Periodical chiral structures were considered in [17]. The transmittance and reflectance of a finite layer with one-dimensional modulation of constitutive parameters have been obtained in this paper using the Kogelnik coupled wave theory [10] generalized to the given case. However, consideration in [17] has been given to a very particular case of normal incidence. As has been shown, in this case chirality and periodicity act in some sense independently. That is, the periodicity leaves qualitative features of the spectrum band structure unaffected, and polarization properties of the output field prove to be qualitatively the same as they are in a homogeneous chiral medium.

In a recent paper [18] the direct numerical method has been applied to bianisotropic media, with constitutive parameters being specified by periodical functions of two space variables. As an example, that paper presents calculated dispersion curves of corrugated layer with bottom ground plane.

In our paper, consideration is given to the general case of oblique wave propagation with respect to the grating vector. To analyze the eigenmode problem in a periodical chiral medium, we use the Bloch function formalism. We show that the joint action of the periodicity and chirality in the general case being considered changes qualitatively both polarization characteristics and the spectrum band structure of the medium.

## II. GENERAL APPROACH

Consider the Bragg diffraction by a one-dimensional periodic chiral medium being characterized by constitutive relations which couple the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ field strengths with the inductions of these fields, $\mathbf{D}$ and $\mathbf{B}$:

$$\begin{aligned}\mathbf{E}&=\eta\eta_{(0)}\mathbf{D}-j\xi c_{(0)}\mathbf{B}\\ \mathbf{H}&=j\xi c_{(0)}\mathbf{D}+\nu\nu_{(0)}\mathbf{B}\ ,\end{aligned} \tag{1}$$

where $\eta_{(0)}$, $\nu_{(0)}$ are the reciprocal permittivity and permeability of an equivalent achiral homogeneous medium, $c_{(0)}=\sqrt{\eta_{(0)}\nu_{(0)}}$ stands for the light velocity in this medium and $\eta$, $\xi$, $\nu$ are the scalar periodic functions of the $z$ coordinate:

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$$\begin{bmatrix}\eta\\ \xi\\ \nu\end{bmatrix}=\sum_{n=-\infty}^{\infty}\begin{bmatrix}\eta_n\\ \xi_n\\ \nu_n\end{bmatrix}e^{-jngz}\ . \tag{2}$$

In this equation $\eta_n,\xi_n,\nu_n$ are given coefficients and $\eta_0,\nu_0=1$. It should be noted that adopted constitutive relations (1) differ in form from the commonly used so-called [19] Lindell-Sihvola (LS), Jaggard-Post-Kong (JPK), and Drube-Born-Fedorov (DBF) equations in that the vectors **D** and **B** are selected as independent variables. Of course, set (1) can easily be reduced by identity transformations to any one of the above mentioned equations. Preference is given to the vectors **D** and **B** because they allow us to present dynamical diffraction equations in the simplest form. However, we must keep in mind that generally the coefficients $\eta_n,\xi_n,\nu_n$ are expressed through all Fourier coefficients of constitutive parameters from the LS, JPK, and DBF equations.

Let us analyze the Maxwell equations under constitutive relations (1) and (2). First, we exclude **E** and **H** from the field equations using relation (1), and then, in accordance with the diffraction theory, represent the vectors $\mathbf{D}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ as the expansions into the Bloch waves,

$$\mathbf{F}(\mathbf{r},t)=e^{-j\omega t}\sum_{n=-\infty}^{\infty}\mathbf{F}_n e^{j\mathbf{k}_n\cdot\mathbf{r}}\ , \tag{3}$$

where

$$\mathbf{F}(\mathbf{r},t)=\begin{bmatrix}\sqrt{\eta_{(0)}}\mathbf{D}(\mathbf{r},t)\\ \sqrt{\nu_{(0)}}\mathbf{B}(\mathbf{r},t)\end{bmatrix}\ ,\quad \mathbf{k}_n=(k_x,0,k_{0z}-ng)\ .$$

$\mathbf{F}_n$ are the six-vectors composed of the unknown constant vectors $\mathbf{D}_n$ and $\mathbf{B}_n$, which are to be determined by appropriate boundary conditions.

Substituting (3) in the Maxwell equations and applying expansion (2) the set of dynamical diffraction equations in chiral media can easily be obtained:

$$\mathbf{k}_n\times\sum_{m=-\infty}^{\infty}\lambda_{n-m}\mathbf{F}_m=k\mathbf{F}_n\ ,\quad n=0,\pm1,\pm2,\ldots, \tag{4}$$

where

$$\lambda_n=\begin{bmatrix}-j\xi_n & -\nu_n\\ \eta_n & -j\xi_n\end{bmatrix}\ ,\quad k=\frac{\omega}{c_{(0)}}\ .$$

Note that the identity $(\mathbf{F}_n\cdot\mathbf{k}_n)=0$ holds true. This fact has motivated the choice of the vectors **D** and **B** as independent variables under the analysis.

Now let us define the polarization vectors of the partial waves by

$$\mathbf{e}_n^{\pm}=\frac{1}{\sqrt{2}}\left[\frac{k_{0z}-ng}{k_n},\pm j,-\frac{k_x}{k_n}\right]\ .$$

The property of the vectors $\mathbf{F}_n$ to be transversal makes it possible to express them in terms of $\mathbf{e}_n^{\pm}$:

$$\mathbf{F}_n=F_n^{+}\mathbf{e}_n^{+}+F_n^{-}\mathbf{e}_n^{-}\ . \tag{5}$$

Then, substituting this expansion into (4), one can obtain the set of scalar equations for partial amplitudes:

$$\left[pj\frac{k}{k_n}I-\lambda_0\right]F_n^p=\sum_{m\neq n}\lambda_{n-m}[(\mathbf{e}_n^{-p}\cdot\mathbf{e}_m^{p})F_m^p+(\mathbf{e}_n^{-p}\cdot\mathbf{e}_m^{-p})F_m^{-p}]\ . \tag{6}$$

The quantity $p$ is equal to $\pm$ if $p$ is an index and $p=\pm1$ if $p$ is a multiplier; $I$ stands for the $2\times2$ unit matrix.

## III. TWO-WAVE APPROXIMATION

General investigation of set (6) is very complicated. To simplify the analysis, we adopt a set of assumptions. First, we restrict ourselves to the case of the harmonically modulated medium, letting $\eta_n,\xi_n,\nu_n=0$ at $|n|>1$. We also suggest that the medium is weakly modulated, that is,

$$|\eta_{\pm1}|,|\xi_{\pm1}|,|\nu_{\pm1}|\ll1\ . \tag{7}$$

Under this assumption, the two-wave approximation [16] can be applied to the diffraction problem analysis. This approximation implies that only the wave with wave vector $\mathbf{k}_1$ (the $Z$ axis is assumed to be oppositely directed with respect to the grating vector **g**) propagates near the Bragg angle and, consequently, only amplitudes $F_{0,1}^{\pm}$ are significant and should be taken into account in set (6). All other diffraction orders have little influence on the diffraction pattern and can be ignored. Thus, in the frame of the two-wave approximation, dynamical diffraction equations in the chiral medium take the form as follows:

$$\begin{aligned}\left[pj\frac{k}{k_0}I-\lambda_0\right]F_0^p&=\lambda_{-1}[(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^{p})F_1^p+(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^{-p})F_1^{-p}]\ ,\\ \left[pj\frac{k}{k_1}I-\lambda_0\right]F_1^p&=\lambda_{1}[(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^{p})F_0^p+(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^{-p})F_0^{-p}]\ .\end{aligned} \tag{8}$$

It is known that there are two different waves with wave vectors $\mathbf{K}^{\pm}$ in a homogeneous chiral medium and

$$K^{\pm}=\frac{k}{1\mp\xi_0}\ ,\quad K_z^{\pm}=\sqrt{(K^{\pm})^2-k_x^2}\ . \tag{9}$$

In view of condition (7), the propagation constants $k_{0,1}$ are little different from $K^{\pm}$. By this reason, the Bragg condition in the chiral medium is split at sufficiently large $\xi_0$ into three independent conditions of the spatial resonance [17] (see Fig. 1):

$$2K_z^{+}=g(1+\delta^{++})\ , \tag{10}$$

$$K_z^{+}+K_z^{-}=g(1+\delta^{+-})\ , \tag{11}$$

$$2K_z^{-}=g(1+\delta^{--})\ . \tag{12}$$

The small corrections $\delta^{ps}$ stand for the resonance detuning. At a given propagation angle, these resonances are realized on different frequencies (Fig. 1). In this case,

$$\delta^{ps}=\frac{\omega-\omega_B^{ps}}{\omega_B^{ps}}\ ,$$

where $\omega_B^{ps}$ stands for the Bragg frequency of the $(p,s)$ res-

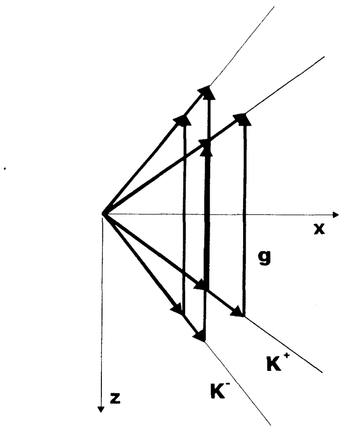


FIG. 1. Graphical representation of all possible resonances in a chiral periodic medium at a given propagation angle. For the sake of definiteness, $\xi_0$ is assumed to be a positive quantity.

onance, which is defined by Eqs. (10)–(12) at $\delta=0$. At a given frequency, the resonances are bound to be observable at different propagation angles or different grating vector lengths.

Precisely, the existence of three independent resonances in periodic chiral media is a main distinctive feature of the problem being analyzed as compared with the diffraction by achiral media.

### A. Model of weakly interacting resonances

Let us assume now that the conditions

$$|\eta_{\pm1}|, |\xi_{\pm1}|, |\nu_{\pm1}| \ll |\xi_0| \tag{13}$$

are satisfied. Physically, they establish the region of the parameters varying where Bragg resonances (10–(12) can be recognized far apart from each other. Since the amplitudes $F_1^{\pm}$ of the diffracted waves are different from zero only in the vicinity of a Bragg resonance, fulfillment of (13) makes is possible to facilitate further analysis by neglecting mixing of the resonances. Conditions (10) and (12) call for the amplitudes $F_{0,1}^{+}$ and $F_{0,1}^{-}$ to be taken into account in Eqs. (8), correspondingly [$(+,+)$ and $(-,-)$ resonances]. In contrast, when condition (11) holds, there are two independent solutions that couple either the amplitude $F_0^+$ with $F_1^-$ [$(+,-)$ resonance] or $F_0^-$ with $F_1^+$ [$(-,+)$ resonance]. So, mathematically, the model of weakly interacting resonance implies that only the variables indicated above are significant in each of the specific cases defined by Eqs. (10)–(12) and should be taken into account under analysis.

Next, impose the restriction on the propagation angle by the condition

$$|\eta_{\pm1}|, |\xi_{\pm1}|, |\nu_{\pm1}| \ll \left[\frac{K_z^{\pm}}{K^{\pm}}\right]^2 , \tag{14}$$

and neglect, in view of (7) and (13), the dependence on $k_{0z}$ of the scalar products involved in (8). Then, eliminating $B_{0,1}^{\pm}$ among Eqs. (8), one can reduce these equations to the form as follows:

$$\begin{aligned} &PD_0^p - \chi^{ps}_{-1} D_1^s = 0 , \\ &\chi_1^{sp} D_0^p + (P-2\delta) D_1^s = 0 . \end{aligned} \tag{15}$$

Here $P=2(K_z^p - k_{0z})/g$. To a first approximation in $|\eta_{\pm1}|, |\xi_{\pm1}|, |\nu_{\pm1}|$, the coupling coefficients $\chi^{ps}_{\pm1}$ are determined by

$$\begin{aligned} &\chi^{ps}_{\pm1} = L_{ps}[\eta_{\pm1} + ps\nu_{\pm1} - (p+s)\xi_{\pm1}] , \\ &L_{ps} = \frac{(K^p)^3}{kgK_z^p}(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^s) . \end{aligned} \tag{16}$$

These coefficients are calculated at zero detuning. Note as well that $L_{ps} \neq L_{sp}$. The quantities $p$ and $s$ in (15) and (16) are varied independently and, thus, these equations define all four possible resonances.

Equations (15) are identical to the equations defining the Bragg diffraction by a periodical achiral medium with the effective coupling coefficients $\chi^{ps}_{\pm1}$ [16]. From the dispersion equation of set (15), one can find nontrivial solutions that establish propagation constants in the medium:

$$k_{0z}^{(1,2)} = K_z^p + \frac{g}{2}\left(-\delta \pm \sqrt{\delta^2 - \chi^{ps}_{-1}\chi^{sp}_1}\right) . \tag{17}$$

Equations (15)–(17) will be used below under the analysis of the eigenmode spectrum and boundary-value problem in the case of weakly interacting resonances.

### B. Model of strongly interacting resonances

Now, we consider the case when the inequality

$$|\xi_0|^2 \ll \max\{|\eta_{\pm1}|, |\xi_{\pm1}|, |\nu_{\pm1}|\} \tag{18}$$

is satisfied instead of (13). Note that (18) does not limit significantly the commonness of the analysis because conditions (13) and (18) taken together cover all the range of the varying parameters $|\eta_{\pm1}|, |\xi_{\pm1}|, |\nu_{\pm1}|$ that is of practical interest. It should also be emphasized that there is a range where (13) and (18) are fulfilled simultaneously. It means that a continuous transition exists from results being given by this model to the results of Sec. III A.

Physically, condition (18) defines the case of closely spaced and, hence, strongly interacting resonances. In this case, all the amplitudes $F_{0,1}^{\pm}$ turn out to be coupled and the approximation used in Sec. III A cannot be applied. Putting the determinant of set (8) equal to zero, we obtain a characteristic equation of the eighth degree in $P$, defined here by

$$P = 1 - \frac{2k_{0z}}{g} .$$

In view of condition (18), $P$ should be taken as being a small quantity. It allows us to neglect the terms $O(P^8)$ and $O(P^6)$ in the characteristic equation, which then takes the form as follows:

$$\Phi(P)=P^4-2(\delta^2+d_0^2-M)P^2+\delta^4-2(d_0^2+M)\delta^2 -4d_0(a_{-1}d_1+a_1d_{-1})\delta+N=0\ . \quad (19)$$

Because resonances (10)–(12) are closely spaced, we have used in (19) the unified Bragg frequency for all of them:

$$\delta=\frac{\omega-\omega_B}{\omega_B}=\frac{2k_z}{g}-1\ ,\quad k_z=\sqrt{k^2-k_x^2}\ .$$

The other parameters in (19) are given by

$$M=a_1a_{-1}+b_1b_{-1}+d_1d_{-1}\ ,$$

$$N=(a_1^2-b_1^2-d_1^2)(a_{-1}^2-b_{-1}^2-d_{-1}^2) -2d_0^2(a_1a_{-1}-b_1b_{-1}+d_1d_{-1})+d_0^4\ ,$$

$$d_0=\xi_0/\cos^2\theta\ ,\quad d_{\pm1}=\xi_{\pm1}\tan^2\theta\ ,$$

$$a_{\pm1}=\tfrac{1}{2}(\eta_{\pm1}+\nu_{\pm1})\tan^2\theta\ ,$$

$$b_{\pm1}=\tfrac{1}{2}(\eta_{\pm1}-\nu_{\pm1})\ ,\quad \tan\theta=k_x/k_z\ .$$

In the next section, we discuss the Floquet-Bloch eigenmodes spectrum, which follows from Eqs. (17) and (19).

## IV. EIGENMODE SPECTRUM

Figures 2–4 show the dispersion characteristics of the periodic chiral medium computed at different values of $\xi_0$

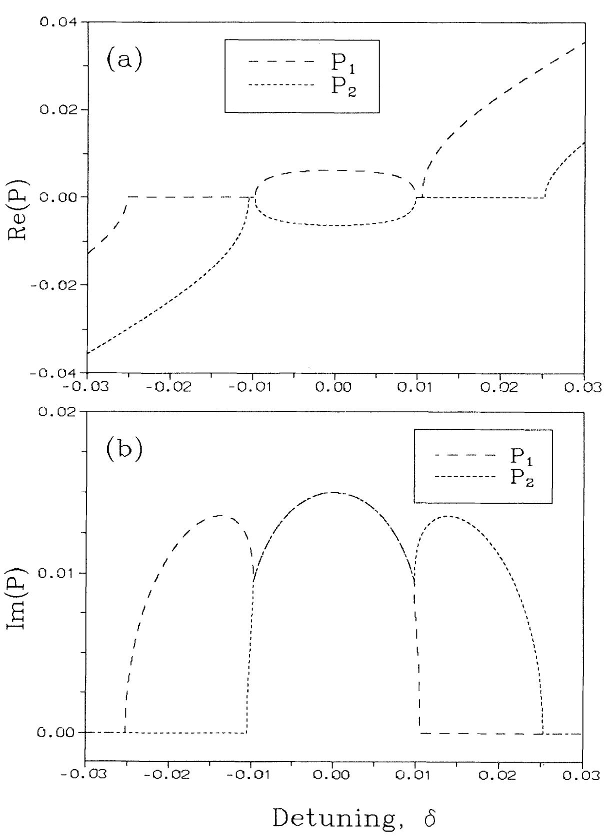


FIG. 2. Eigenmode spectrum in the case of strongly interacting resonances. $\xi_0=0.0065$, $\eta_{\pm1}=0.03$, $\nu_{\pm1}=0$, $\tan\theta=0.7$.

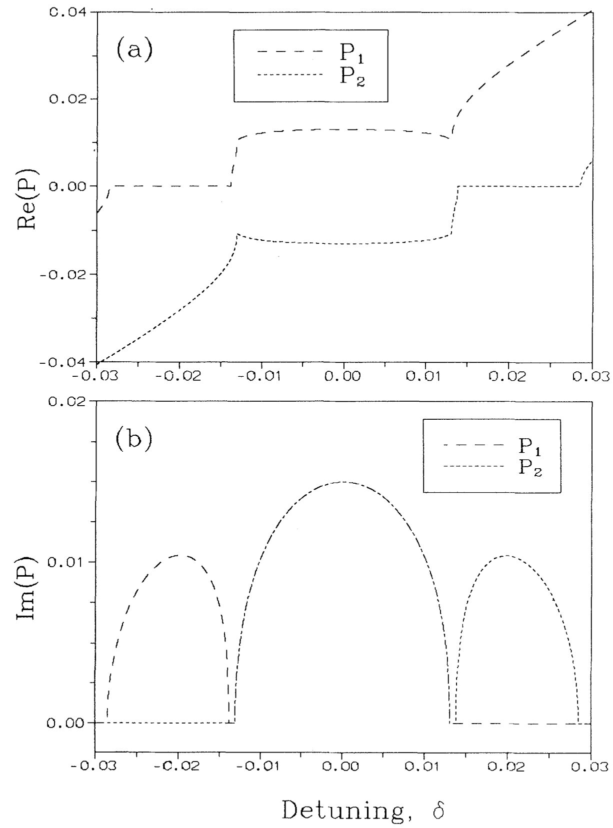


FIG. 3. Transformation of the eigenmode spectrum under effect of the chiral admittance growth. $\xi_0=0.01$; all other parameters are the same as those in Fig. 2.

in the case when $\xi_{\pm1}=\nu_{\pm1}=0$. So far as $\xi_0$ is assumed to be a small quantity, such conditions correspond to the medium with the weakly modulated permittivity, constant permeability, and chiral admittance. The case of the conventional Bragg diffraction by the achiral medium is presented by dashed lines in Fig. 4. Two eigenmodes ex-

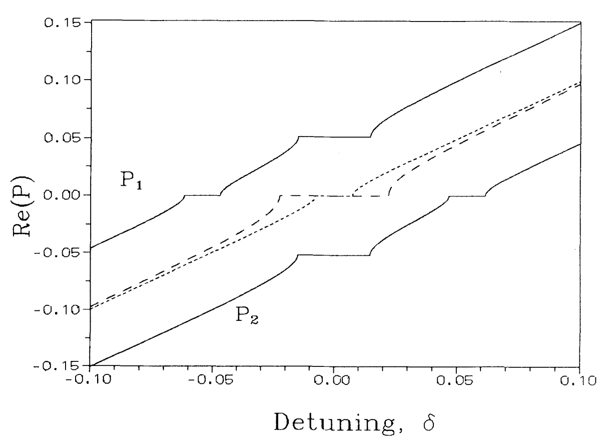


FIG. 4. Eigenmode spectrum in the case of weakly interacting resonances. $\xi_0=0.035$; all other parameters are the same as those in Fig. 2. Dashed lines correspond to the equivalent achiral medium.

isting in this case are linearly polarized and have different forbidden band gap widths.

The appearance of the chirality results in a qualitative change of the dispersion characteristics. The curves in Fig. 2 illustrate the case of strongly interacting resonances. In the interior of the central forbidden band gap, the dependence Re($P$) on $\delta$ [Fig. 2(a)] forms a closed loop that intersects the axis Re($P$)=0 in the points of Jordan multiplicity ($J$ multiplicity) of the matrix operator (8). In these points this operator becomes a nondiagonalized one containing the $2\times 2$ Jordan canonical blocks. Analytically, the points of $J$ multiplicity appear where Eq. (19) and the condition $d\Phi/dP=0$ are satisfied simultaneously. Expressing from the latter $P$ in terms of $\delta$ and substituting into (19), one can find the values of $\delta$ corresponding to the points of $J$ multiplicity:

$$\delta_{1,2}=-\frac{1}{2d_0}\left[a_1d_{-1}+a_{-1}d_1\pm\sqrt{(a_1d_{-1}+a_{-1}d_1)^2+N-(d_0^2-M)^2}\right] . \tag{20}$$

So far as a physical meaning may have only real vales of $\delta$, the condition

$$(a_1d_{-1}+a_{-1}d_1)^2>(d_0^2-M)^2-N , \tag{21}$$

which follows from (21), defines the range of the constitutive parameters conforming to the loop formation.

In the points of $J$ multiplicity ($\delta=\delta_{1,2}$), the associated waves occur with amplitudes linearly dependent on the spatial coordinate. These waves are expressed in terms of the associated Keldysh functions [20] forming, along with the eigenfunctions, the system of root functions of the non-self-adjoint operator of the problem being considered. The associated waves were studied in certain types of microwave waveguides [21–23] and in the crystal optics (Voight [4] and Petrov–Fedorov [24,25] waves). In the given case, it is remarkable that the associated waves appear, owing to the interaction of resonances (10)–(12) and, consequently, they are a result of the mutual influence of periodicity and chirality.

The loop on dispersive characteristics existing at $\delta_2<\delta<\delta_1$ is the band of ambiguous dispersion with anomalous dispersion in the range where $d\,\mathrm{Re}(P)/d\delta<0$. These features are identical to the effect of complex waves in certain types of regular waveguides [26,27].

The dependence Im($P$) on $\delta$ is also of interest. When $\xi_0$ is small, the plot of this dependence [Fig. 2(b)] can be referred to as a "closed bud." As $\xi_0$ increases, the bud opens [Fig. 3(b)] and the loop pulls apart [Fig. 3(a)]. The "petals" of this bud move apart in the process and intervals of transparency appear between them. The central petal (forbidden band gap) is opaque to both eigenmodes, whereas the side band gaps are opaque to only one of them. The further growth of the chirality results in transparent intervals broadening and, consequently, in the depression of resonance interaction. It leads gradually to the case of weakly interacting resonances (Fig. 4), being defined by condition (13). Eigenmodes in this case are circularly polarized, that is, the inequality (13) is the condition of separation of different polarization states.

Thus, in the chiral medium at sufficiently large admittance $\xi_0$, the central forbidden band gap, which is common to both waves, is accompanied by two satellites. Their position with respect to the central bandgap is determined by conditions (10) and (12). At $\xi_0>0$ the satellites for left- and right-circularly polarized waves are above and below the central band gap, correspondingly. At $\xi_0<0$ the situation is reversed.

The satellites decrease in width as the propagation angle grows smaller, and they fully disappear when the vectors $\mathbf{K}^p$ become collinear with the grating vector $\mathbf{g}$. In this particular case, the amplitudes $\mathbf{F}_0^p$ and $\mathbf{F}_1^p$ turn out to be uncoupled in (8) due to $(\mathbf{e}_0^{-p}\cdot\mathbf{e}_1^p)=0$ and, consequently, resonances (10) and (12) can in no way be manifested. This fact was revealed in [17]. Thus, our results at $\mathbf{K}^p\|\mathbf{g}$ are in agreement with the consideration given in [17]: the band structure of the spectrum is analogous to that in achiral media, but it conforms to circularly polarized waves.

The situation is opposite that considered above in the medium with the modulated chirality $\xi$ and constant $\eta$ and $\nu$. As follows from (16), both resonances (11) do not manifest themselves under an arbitrary propagation angle. As a result, the central band gap disappears in the spectrum under given conditions.

## V. BOUNDARY-VALUE PROBLEM

Let a circularly polarized plane wave $\exp(j\mathbf{K}^p\cdot\mathbf{r})\mathbf{e}_0^p$ be incident obliquely at a layer of periodically modulated chiral medium bounded by the equivalent homogeneous medium with the parameters $\eta_0,\xi_0,\nu_0$. The layer boundaries are considered to be perpendicular to the $Z$ axis. We restrict ourselves to the model of weakly interacting resonances studied in Sec. III A. Then, to this approximation, the electric field inside and outside the layer near the $(p,s)$ resonance can be written in the form as follows:

$$\begin{aligned}
&\mathbf{D}_{\mathrm{in}}(\mathbf{r})=e^{j\mathbf{K}\cdot\mathbf{r}}\mathbf{e}_0^p+D_r^s e^{j\mathbf{K}_r^s\cdot\mathbf{r}}\mathbf{e}_1^s ,\\
&\mathbf{D}_{\mathrm{out}}(\mathbf{r})=D_{\mathrm{out}}^p e^{j\mathbf{K}^p\cdot\mathbf{r}}\mathbf{e}_0^p ,\\
&\mathbf{D}(\mathbf{r})=[D_{01}^p e^{j\mathbf{k}_0^{(1)}\cdot\mathbf{r}}+D_{02}^p e^{j\mathbf{k}_0^{(2)}\cdot\mathbf{r}}]\mathbf{e}_0^p\\
&\qquad+[D_{11}^s e^{j(\mathbf{k}_0^{(1)}+\mathbf{g})\cdot\mathbf{r}}+D_{12}^s e^{j(\mathbf{k}_0^{(2)}+\mathbf{g})\cdot\mathbf{r}}]\mathbf{e}_1^s ,
\end{aligned} \tag{22}$$

where $\mathbf{K}_r^s=(k_x,0,-K_z^s)$, $\mathbf{k}_0^{(i)}=(k_x,0,k_{0z}^{(i)})$. The wave numbers $k_{0z}^{(i)}$ are determined from (17). We neglect in (17) the effect of reflection because the constitutive parameters $\eta_0,\xi_0,\nu_0$ are assumed to be continuous at the layer boundaries. It makes possible scalar formulation of boundary conditions for the fields defined by Eqs. (22):

$$\begin{aligned}
&1=D_{01}^p+D_{02}^p , \quad D_r^s=D_{11}^s+D_{12}^s ,\\
&D_{\mathrm{out}}^p e^{jK_z^p l}=D_{01}^p e^{jk_{0z}^{(1)}l}+D_{02}^p e^{jk_{0z}^{(2)}l} ,\\
&0=D_{11}^s e^{jk_{0z}^{(1)}l}+D_{12}^s e^{jk_{0z}^{(2)}l} ,
\end{aligned} \tag{23}$$

where $l$ is the layer thickness.

Note that $D_{0i}^{p}$ and $D_{1i}^{s}$ are the linearly dependent solutions of set (15) corresponding to the solution $k_{0z}^{(i)}$ (17) of the characteristic equation. Thus, Eqs. (23) allow the determination of all unknown amplitudes inside and outside the layer.

As might be expected from the diffraction problem formulation in the case of weakly interacting resonances, boundary conditions (23) as well as diffraction equations (15) turn out to be identical to the corresponding equations for the Bragg diffraction by a periodical achiral medium [16]. In view of this fact, one can use well-known expressions for field amplitudes given, for instance, in [16]. In particular,

$$D_r^s = \frac{\chi_1^{sp}}{\delta + j\sqrt{\delta^2 - \chi_{-1}^{ps}\chi_1^{sp}}\cot\left[\frac{lg}{2}\sqrt{\delta^2 - \chi_{-1}^{ps}\chi_1^{sp}}\right]} . \quad (24)$$

Note that $D_r^s$ as well as other amplitudes in (24) depend on both indices $p$ and $s$. We use only one of them corresponding to the polarization of a given wave to avoid cumbersome designations.

For an incident wave of unit amplitude the quantity $D_r^s$ expresses the reflection coefficient with respect to **D**. In general, this coefficient does not coincide with the reflection coefficient relative to **E**. Indeed, by extracting **E** from the constitutive relations (1), one can obtain, through some manipulations, that $E_r^s$ is given by formula (24), in which the coupling coefficients $\chi_{-1}^{ps}, \chi_1^{sp}$ (16) are replaced by

$$\tilde{\chi}_{-1}^{ps} = \frac{K^s}{K^p}\chi_{-1}^{ps} , \quad \tilde{\chi}_1^{sp} = \frac{K^p}{K^s}\chi_1^{sp} . \quad (25)$$

It can easily be shown that in lossless chiral media, as distinct from conventional dielectrics, $|D_r^s|^2, |E_r^s|^2 \neq 1$ near the $(p, -p)$ resonance (central forbidden band gap) even at $l \to \infty$. Moreover, it can easily be shown that $|D_r^s|^2, |E_r^s|^2 > 1$ at certain conditions. The equality $|E_r^s|^2 = 1$ holds true only at the normal incidence, that is in agreement with [17]. In this case, too, $|D_r^s|^2 \neq 1$. It should be emphasized that the inequality $|D_r^s|^2, |E_r^s|^2 \neq 1$ does not contradict energy conservation, as might appear at first sight. The quantities $D_r^s, E_r^s$ defined by Eqs. (24) and (25) couple different modes propagating at different angles with respect to grating boundaries. That is why a conventional statement of conservation of energy under the Bragg diffraction cannot be applied to the case being studied. The situation here is analogous to that in plasma waveguides under the eigenmode reflection from transverse metal walls [28] where $|E_r^s| > 1$ as well. As has been mentioned in [6], from this point of view it would be more correct to refer to $D_r^s, E_r^s$ as transformation coefficients instead of reflection ones.

Equation (24) holds true in the vicinity of the Bragg frequency $\omega_B^{ps}$. As $\omega$ recedes from the edges of a corresponding forbidden band gap, being defined by the equation $|\delta| < \sqrt{\chi_{-1}^{ps}\chi_1^{sp}}$, the coefficient $D_r^s$ rapidly decreases. For this reason, expression (24), being applicable in each case only near a corresponding resonance, can be extended beyond resonance regions. It enables the unified presentation of the reflection field over the entire frequency region of two-wave diffraction. Let an incident field be given by

$$\mathbf{D}_{\text{in}}(\mathbf{r}) = \sum_{p=\pm 1} D_{\text{in}}^{p} e^{j\mathbf{K}^p\cdot\mathbf{r}}\mathbf{e}_0^p , \quad (26)$$

where $D_{\text{in}}^p$ are given quantities. Then, taking into account the above reasoning, the reflected field can be presented in the following form:

$$\mathbf{D}_r(\mathbf{r}) = \sum_{p,s=\pm 1} D_{\text{in}}^{p} D_r^s e^{j\mathbf{K}^s\cdot\mathbf{r}}\mathbf{e}_1^s . \quad (27)$$

Recall that the amplitudes $D_r^s$ depend on both indices.

Equation (27) shows that, generally, the layer being considered has three reflection band gaps. The central one reflects the fields of both polarization, whereas side band gaps are selective with respect to the polarization sign. This is a distinguishable feature of the Bragg diffraction in chiral media.

## VI. CONCLUSION

We have presented a theory for describing Floquet-Bloch wave diffraction by periodically inhomogeneous chiral media at arbitrary mutual orientation of the propagation and grating vectors. The theory predicts novel dispersive properties of such media that are qualitatively different from those in the particular case of the collinearly oriented vectors mentioned above. It is of interest that these properties prove to be similar in many respects to dispersive characteristics being manifested by magnetically ordered Mössbauer crystals and cholesteric liquid crystals [13]. This analogy supports a relationship between the optical properties of such crystals and chiral periodic media.

It should be emphasized that in the case of weakly interacting resonances, when a transparent interval between the central band gap and the satellite is sufficiently large, the chiral periodic medium can be considered as equivalent to an achiral grating with the same periodicity and an effective permittivity. This permits the extension of the conventional diffraction theory results [10,16] to the case being studied. We have used it in Sec. V, analyzing the plane wave diffraction by a chiral periodic layer.

Such an approach can also be applied to the investigation of nonstationary process in chiral gratings. The problem of the pulse propagation through a chiral medium is of great interest, but this problem has been analyzed in the simplest case of the chiral homogeneous layer [29]. On the other hand, a rigorous time-domain analysis of an ultrashort pulse diffraction by dielectric gratings has been given in [30,31]. In the case of weakly interacting resonances, this time-domain analysis is applicable to the chiral periodic layer with effective coupling coefficients (16). By analogy with [30,31], one can predict the effect of temporal compression with respect to circularly polarized phase-modulated pulses.


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